11620. *Proposed by Mathew Rogers, Université de Montréal, Montreal, Canada.* Let H_k be the kth Hermite polynomial, given by $H_k(x) = (-1)^k e^{x^2} \frac{d^k}{dx^k} e^{-x^2}$. Suppose

$$\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_1+\rho_1} & \frac{1}{\rho_1+\rho_2} & \cdots & \frac{1}{\rho_1+\rho_M} \\ \frac{1}{\rho_2+\rho_1} & \frac{1}{\rho_2+\rho_2} & \cdots & \frac{1}{\rho_2+\rho_M} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\rho_M+\rho_1} & \frac{1}{\rho_M+\rho_2} & \cdots & \frac{1}{\rho_M+\rho_M} \end{pmatrix} \begin{pmatrix} \frac{1}{\rho_1} \\ \frac{1}{\rho_2} \\ \vdots \\ \frac{1}{\rho_M} \end{pmatrix},$$

where $\rho_1, \ldots, \rho_M$ are complex numbers for which $\sum_{k=1}^{M} 1/\rho_k > 0$. Prove that each ρ_k is a root of the equation

$$H_M(ix) - i\sqrt{2M}\,H_{M-1}(ix) = 0.$$